Some Numerical Simulations for Educational Tools Using Personal Computer

—Tsunami, Seismic Wave, Pyroclastic Flow, Erosion etc.—

Yoshio Okamoto (Osaka Prefectural Education Center, Japan)

1. Preface

Around the Circum-Pacific region, many disasters such as earthquakes, tsunamis or eruptions of volcanoes have been caused to relate with the plate motion. We developed some PC based numerical simulations to explain the mechanism of these events for the teaching tools. Several prototypes written in BASIC language for the NEC-PC98 (Japanese domestic PC) have been translated to C language on Linux, which allows to run on the IBM-PC (or compatible) with higher speed and with more extended grid area. The result of calculation can be visualized on CRT at each stage, using Xlib graphics. In this paper we describe their methods, features and some extensions to other field.

2. Tsunami Simulation

Tsunami is the most successful example of the numerical simulation in geophysics. We developed a numerical simulation to calculate the tsunami propagation under both arbitrary bathymetry and arbitrary initial condition. The program code is introduced on PC simplifying the one for the workstation or the mainframe computer. Basic equations with finite difference scheme (Abe et.al.,1991) are as follows,

Equation of motion:

$$u(i,j,k)=u(i,j,k-1)-g*dt/ds*\{w(i,j,k)-w(i-1,j,k)\}$$

$$v(i,j,k)=v(i,j,k-1)-g*dt/ds*\{w(i,j,k)-w(i,j-1,k)\}$$

Equation of continuity:

$$w(i,j,k)=w(i,j,k-1)$$
$$-dt/ds*\{hu(i+1,j)*u(i+1,j,k-1)-hu(i,j)*u(i,j,k-1)$$
$$+hv(i,j+1)*v(i,j+1,k-1)-hv(i,j)*v(i,j,k-1)\}$$

where u, v are x and y components of flow velocity, w is the wave height, hu, hv are the water depth on staggered grid, g is the gravity acceleration. i, j denote the grid number, k denotes the time step, dt, ds are the step interval of time and space, respectively .

Boundary condition: bathymetric data (1km mesh or 5 km mesh) is used. The transparant condition is employed on the grid edge to suppress reflected “ghost” waves.

Initial condition: Gaussian function or vertical lift of sea surface calculated from the fault model is assumed . Fig.1 shows an example.

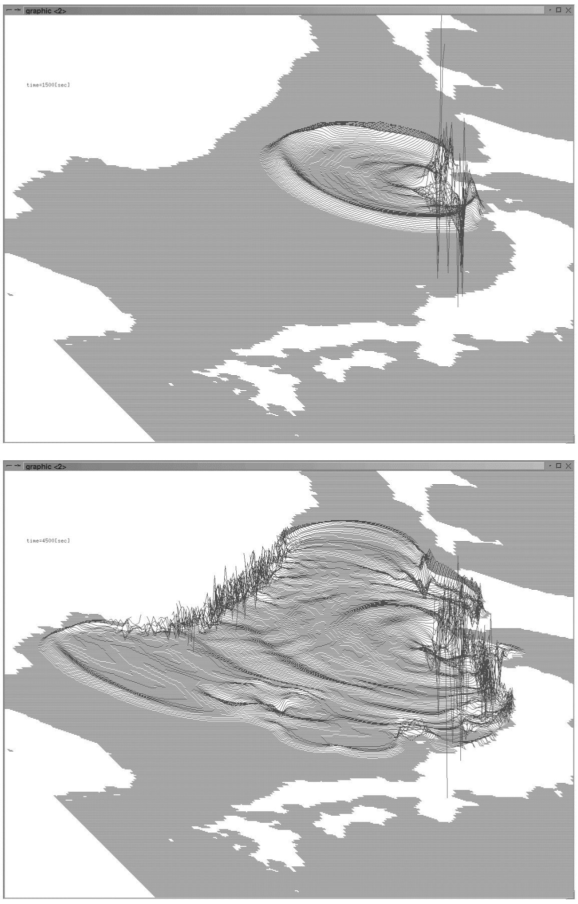

Fig.1 Tsunami propagation of 1993 Hokkaido Nansei-oki earthquake (more than 200 people were killed)

3. Seismic Wave Propagation

The seismic wave propagation has very

important role in geophysics. Our program simulates the seismic wave propagation solving the finite difference SH-wave equation under heterogeneous condition, which is defined as the variation of elastic constant of subsurface materials. In this section, we treat only two examples, i) The cause of "disaster belt" (heavy damage zone) of the 1995 Kobe earthquake (Fig.2-a), and ii) The head waves returning back from the Moho discontinuity (fig.2-b).

Equation of motion (2-dimensional SH-wave equation with finite difference scheme [Aki and Richards, 1980]):

$$\begin{aligned} &v(i,j,k+1)=2*v(i,j,k)-v(i,j,k-1)\\ &\quad +(dt/ds)^2/d(i,j)*\{m(i+1/2,j)*v(i+1,j,k)\\ &\quad +m(i-1/2,j)*v(i-1,j,k)+m(i,j+1/2)*v(i,j+1,k)\\ &\quad +m(i,j-1/2)*v(i,j-1,k)-v(i,j,k)*\{m(i+1/2,j)\\ &\quad +m(i-1/2,j)+m(i,j+1/2)+m(i,j-1/2)\} \end{aligned}$$

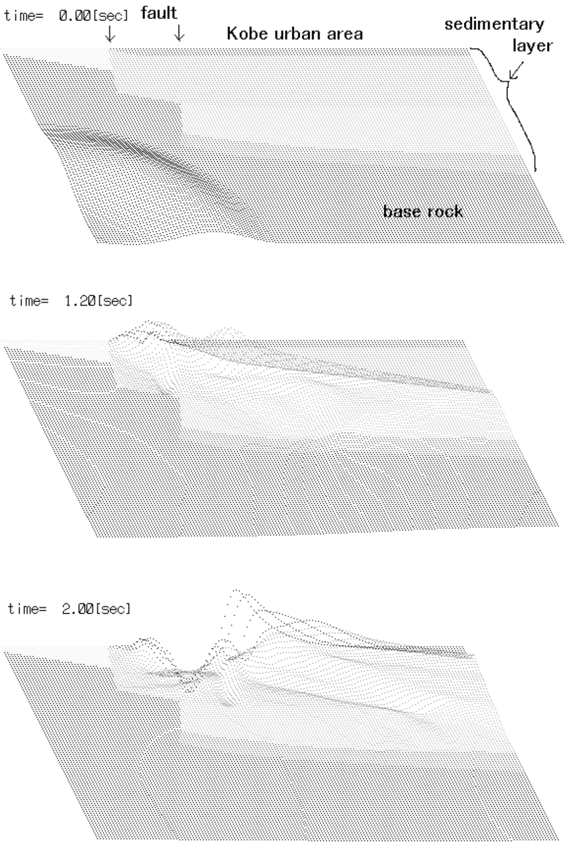

Fig.2-a "Disaster Belt" of 1995Kobe earthquake
N-S cross section of Kobe urban area.
Huge waves developed at the surface by wave refraction and interference .

where $d(i,j)$ is the density of the medium, $m(i,j)$ is the rigidity. v is the displacement in the y-direction, i, j denote the grid index, k denotes the time step, dt, ds are the interval of time and space.

Boundary condition: $d(i,j)$ and $m(i,j)$ are derived from $Vs(i,j)$(shear wave velocity) assumed as substructure. In the model of Kobe, Vs of sediment layers decreases gradually toward the ground surface (Pitarka, 1996). In the model of Moho, Vs abruptly changes at the crust/mantle boundary. In both cases, the scale of parameters is quite simplified for visual modeling.

Initial condition: A simple sine wave source is assumed.

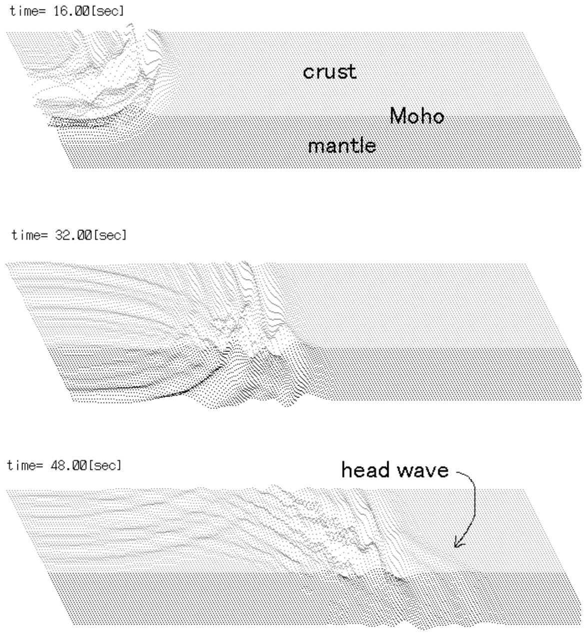

Fig.2-b Head waves from "Moho discontinuity"
Vs: crust (light gray) < mantle (dark gray)

4. Pyroclastic Flow

Since volcanic eruption shows many complex behaviors, it is difficult to evaluate or to anatomize the mechanism of eruption. Especially pyroclastic flow is common in acid volcano, but the detail of its mechanism is still obscure. In this section, we introduce a new simple kinetic model called "descending ball model" (Okamoto, 1998), in which the pyroclastic flow is treated as a ball placed on the lava dome and descends along the slope of volcano. This model is an original extension of the erosion model (see section.5). ***The basic algorithm and outlines*** are as follows,

i) The digitized volcano map in which the height h(i,j) is defined at all grid is used to calculate the descending trace of a pyroclastic flow (This map was created by our high school students).

ii) First, a "ball" is placed on the lava dome site s(i,j), which descends down to the lowest site among 8 adjacent neighbors.

iii) This process is repeated until the ball reaches the sea or the lower site surrounded by hills.

iv) If the ball is stopped or reached the sea, then a next ball starts at the lava dome site.

In this algorithm, the ball should fall down always through the same trace, so our model introduced some ***stochastic effects*** as follows,

i) Add randomness to select the start site. *s(i,j)* is changed by *s(i+rnd,j+rnd);rnd* is a random value.

ii) Add inertial force effect to select fall direction. If the course should turn to left or right because of local gradient, sometimes ball keeps inertial direction.

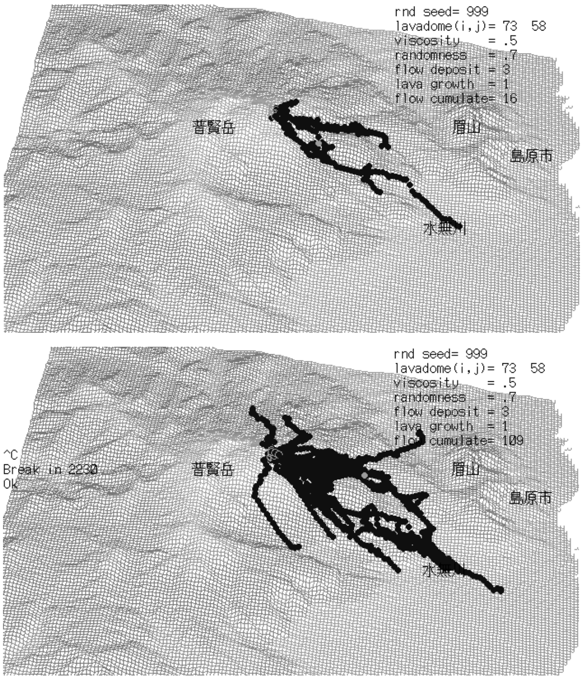

Fig.3 An example of our pyroclastic flow model of Unzen volcano eruption in 1991 . Upper: early stage Lower: late stage The traces are close to the real case.

iii) Add friction between the ball and the slope. The friction is inversely related to the local gradient of slope. If the random number generated by PC exceeded a threshold value, then the ball stops at the site by "friction".

These additional characters allow to increase randomness and fluctuation, therefore the trace of “balls” can simulate the real pyroclastic flows. Fig.3 simulate the pyroclastic flows at Unzen volcano (Kyusyu, Japan) eruption in 1991.

5. Erosion

Recently, the simulation of landscape formation is discussed especially in relation to the “cellular automaton” or the “fractal” analysis. In this section, a simple erosion model (Boger et.al., 1991) is introduced and modified. The erosion control parameters are chosen and evaluated. ***The basic algorithm and outlines*** are as follows,

i) The initial land surface is represented by square lattice (grid) with heights *h(i, j)* .

ii) A "rain drop" falls at a site where is randomly selected on the lattice, and flows down to the lowest neighbor site.

iii) The lattice site passed by rain is either eroded or filled by sediments depending on the local gradient, so that the landform changes gradually. The height *h1* from which rain dorp moves is decreased by *de*(h1-h2)* for erosion, and the height *h2* which the drop moves to is increased by *ds*(h1-h2)* for deposit of sediment. We called "*de*" as the erosion parameter and "*ds*" as the sediment or deposit parameter. *de* and *ds* are assumed in range of 0.0 -0.5. They are usually constant, but allow to change depending on time or local gradient.

iv) This process is iterated until the rain drop reaches the “sink” which is a river mouth of the lattice. And then a new raindrop falls down. If the rain drop reaches the end of lattice except the sink, the periodic boundary condition is applied.

During these processes, the erosion parameter *de* and the deposit parameter *ds* play very important role for the landscape evolution. If *de=ds* (mass conservation model), then the landscape evolution is simulated as the "diffusion model (Culling, 1965)" (Fig.4-a). This model is suitable to explain the river network evolution, but the rugged profile will be lost gradually with time. In contrast, if *de>ds* (mass conservation break model), the landscape will be eroded rapidly like the Grand Canyon (fig.4-b). More precise study of these parameters is now under investigation.

Now, we extend our model to involve the crustal movement. These effects can be applied by lifting or shifting a block of lattice during the calculation. In this way, some active fault landscapes such as the offset river, the spur facet of mountain end are derived (see Fig.5-a,b).

6. To the Future

Recently, PC's potential of calculation and visualization has become powerful. Furthermore, new algorithms are derived in geoscience from the thinking of "complexity" such as "fractal" or "cellular automaton". Therefore we continue to develop and improve numerical simulations for the field which is difficult to simulate by old analog experiments. One of our goal is to create "another virtual earth" in PC, which fascinates and amuses the students just like "Disney Land"!

Acknowledgements;

We appreciate Mr. Dinesh Pathak for his useful comment in English.

References:

Abe K. and E. Noguera B. : *Bull. of Nippon Dental Univ.Gen.Edu.*, **21**, 25- 38, 1992

Aki K.and P.G.Rchards: *Quantitative Seismology,* 781, 1980,

Culling W.E.H.: *J. geol.*, **73**, 230-254, 1965

Meakin P.: *Rev. of Geophysics*, **29**, 317-354, 1991(including [Boger et.al., 1991] as a personal communication)

Okamoto Y.: *Educat.Earth Sci.*, **51**(3), 97-105, 1998 (in Japanese)

Okamoto Y.: *Educat.Earth Sci.*, **52**(2), 53-62, 1999 (in Japanese)

Pitarca A.: *J. Phys Earth,* **44**, 563-576, 1996

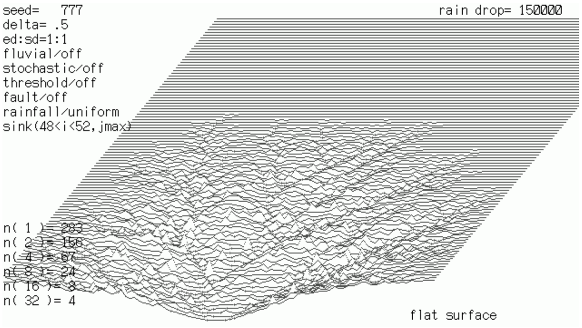

Fig.4-a Mass conservation model (de=ds=0.5)

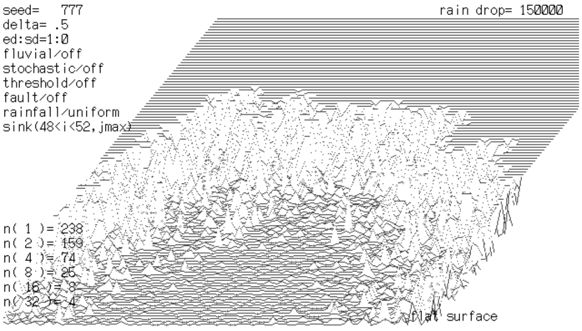

Fig.4-b Grand Canyon model (de=0.5,ds=0)

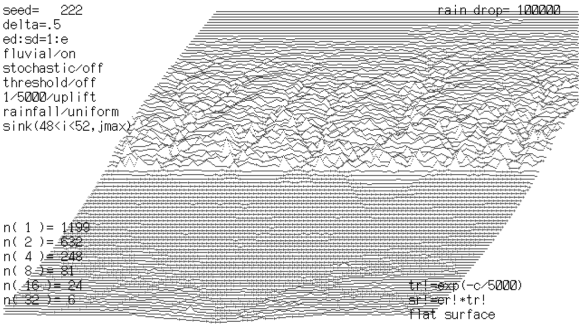

Fig.5-a Vertical uplift model (spur-end facet and confluent fan are formed)

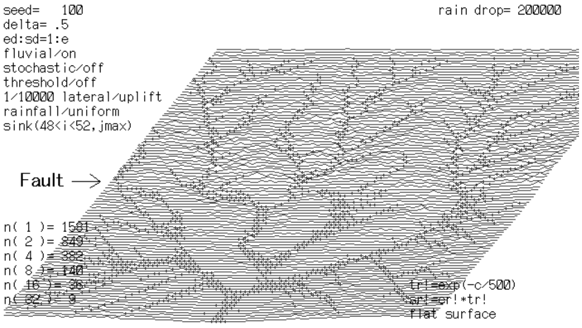

Fig.5-b Lateral fault model (left offsets of rivers are formed)